Thin-film superconductor-ferromagnet hybrids: Competition between nucleation of superconductivity at domain walls and domains' centers

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By using the Ginzburg-Landau theory we studied the order parameter nucleation in the planar superconductor-ferromagnet (S/F) hybrids of a finite thickness, when the magnetic field is inhomogeneous both in the lateral and transverse directions. We calculated the dependence of the critical temperature T_c on the external magnetic field H and studied the transformation of the phase transition line $T_c(H)$, induced by the variation of the parameters of the S/F bilayers such as the period of the domain structure $2L$ and the thicknesses D_s and D_F of the superconducting and ferromagnetic films. The actual shape of the $T_c(H)$ line is demonstrated to be determined by the nucleating regime at $H=0$. For the thin-film S/F hybrids $(L/D_F \gg 1)$ a possible type of the phase boundary $T_c(H)$ is predicted–the reentrant superconductivity with the positive slope $dT_c/d|H|$ at $H=0$, which appears as a consequence of the order parameter nucleation at the centers of the magnetic domains at $H=0$. The nucleation near the domain walls at $H=0$, resulting in the $T_c(H)$ line with zero slope $dT_c/dH|_{H=0}$, can be stimulated, e.g., by increasing the D_s value or decreasing the ratio L/D_F.

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I. INTRODUCTION

Superconductor-ferromagnet (S/F) hybrids have recently attracted considerable attention due to a nontrivial interaction between superconductivity and ferromagnetism, which leads to the appearance of unusual physical phenomena [see the reviews[1–3] and references therein]. In particular, an inhomogeneous magnetic field, induced by the ferromagnet, affects strongly the nucleation of the superconducting order parameter (OP). As a consequence, the S/F hybrids have quite an exotic T-H diagram and the corresponding phase transition line $T_c(H)$ differs significantly from the standard linear dependence, typical for bulk superconducting samples in a uniform magnetic field H,

$$\frac{T_c}{T_{c0}} = 1 - \frac{|H|}{H_{c2}^0}, \quad (1)$$

where T_{c0} is the critical temperature of the superconducting transition in a zero magnetic field, $H_{c2}^0=\Phi_0/(2\pi\xi_0^2)$ and ξ_0 are the upper critical field and the coherence length at $T=0$, Φ_0 is the magnetic flux quantum. The OP nucleation in various S/F hybrid systems as well as the shape of the $T_c(H)$ line were analyzed theoretically[4–8] and studied experimentally:[9–17] for superconducting films and nanostructured superconductors with ferromagnetic dots[9–13] and for superconducting films deposited on ferromagnetic substrates.[14–17]

In this paper we focus on the OP nucleation in hybrid planar structures, consisting of a large-area superconducting (S) film, separated from a ferromagnetic (F) film by a thin insulating layer (see Fig. 1). We assume that there is a stripe-type domain structure inside the F film and the out-of-plane magnetization $\mathbf{M}=M_z(x)\mathbf{z}_0$ changes the sign periodically along the x axis with the period $2L$, keeping the absolute value of the magnetization M_0 unchanged (here the z axis is taken perpendicular to the F film, L is the domain width). Here we can generalize the recent theoretical results[4,5,16] for the S/F systems, consisting of the S and F films of a finite thickness. As a result, the stray magnetic field inside the S film is a periodic function along the x coordinate (in the lateral direction) and a monotonously decreasing function along the z coordinate (in the transverse directions). We will show, that only the lateral inhomogeneity is responsible for the appearance of the $T_c(H)$ dependence typical for the reentrant superconductivity, while the transverse inhomogeneity masks this effect.

The pronounced lateral inhomogeneity of the magnetic field, especially for thin-film S/F hybrids, gives new possibilities for the OP manipulation using "magnetic templates" and leads to the different regimes of the OP nucleation. We predict that at $H=0$ the superconducting nuclei can appear at the regions near the $|b_z(\mathbf{r})|$ minima even if the transverse component of the magnetic field $b_z(\mathbf{r})$ vanishes locally. Earlier this scenario was realized only for $|H| \gtrsim B_0$ (see Refs. 5, 6, and 16), where the compensation field B_0 is of the order of the z component of the field in the centers of magnetic domains. We expect that the shift of the superconducting nucleus between the centers of domains with the positive and negative magnetization at the sign inversion of H is respon-

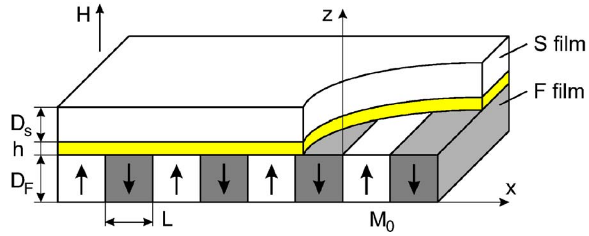

FIG. 1. (Color online) The schematic presentation of the planar bilayer S/F structure, where L is the width of domains, D_s and D_F are the thicknesses of superconducting and ferromagnetic films, h is the insulating spacer thickness.

sible for an unusual phase boundary $T_c(H)$ with the *positive slope* $dT_c/d|H| \simeq T_{c0}/H_{c2}^0$ at $H=0$, which was not yet found theoretically or experimentally. By decreasing the ratio L/D_F or by increasing the D_s or h values one can suppress the OP nucleation in the centers of domains and promote the OP nucleation above domain walls at $H\simeq 0$ (here D_F and D_s are the thicknesses of the F and S films, and h is the thickness of an insulating buffer layer between F and S films). Such modification of the nucleating regime should result in the appearance of the parabolic behavior of the $T_c(H)$ line at low H values (i.e., $dT_c/dH|_{H=0}=0$).

Here we present the results of the numerical solution of the two-dimensional (2D) linearized Ginzburg-Laudau equation. We demonstrate how the OP nucleation in the considered S/F systems can be treated in the framework of the simplified perturbed one-dimensional (1D) equation. We also present some analytical estimates, which are valid in several important cases. These universal asymptotic forms provide a deeper insight into this problem and make it possible to evaluate the influence of the nonuniform magnetic field on the $T_c(H)$ line before doing any detailed calculations. We hope that the present study is of particular importance for a better understanding of the OP nucleation in the presence of an inhomogeneous magnetic field and for the practical realization of the S/F hybrid systems for various applications.

II. MODEL

We analyze the problem of the order parameter (OP) nucleation in S/F hybrid structures, based on the linearized Ginzburg-Landau (LGL) equation in the given magnetic field $\mathbf{B}(\mathbf{r})=\mathbf{b}(\mathbf{r})+H\mathbf{z}_0$:

$$-\left[\nabla + \frac{2\pi i}{\Phi_0}\mathbf{A}(\mathbf{r})\right]^2\Psi = \frac{1}{\xi^2(T)}\Psi. \quad (2)$$

Here $\Psi(\mathbf{r})$ is the OP distribution, $\mathbf{A}(\mathbf{r})$ is the vector potential, and $\mathbf{B}(\mathbf{r})=\mathrm{rot}\,\mathbf{A}(\mathbf{r})$ is the total magnetic field; $\mathbf{b}(\mathbf{r})$ is the stray field, induced by the domains in the F film; H is the external magnetic field, oriented perpendicular to the plane of the structure; $\Phi_0=\pi\hbar c/e$ is the flux quantum, $\xi(T)=\xi_0/\sqrt{1-T/T_{c0}}$ is the coherence length, $i^2=-1$. Note that in Eq. (2) we neglect the corrections to the vector potential, caused by the screening currents, since the supercurrents are proportional to $|\Psi|^2$ and this results in higher-order terms, omitted in the linear approximation.[18] We assume that the S and F films are spatially separated by an insulating layer of the thickness $h\neq 0$ (see Fig. 1), therefore the proximity effect can be neglected in comparison with the electromagnetic (orbital) effect. Different aspects of the exchange interaction in the S/F hybrids were considered in Refs. 1–4 and 17.

We consider here the case, when the inhomogeneous magnetic field is induced by the F film of the thickness D_F with the 1D step-like distribution of magnetization

$$M_z(x) = M_0\,\mathrm{sgn}(x) \quad \text{for } |x|<L,$$

$$M_z(x-2Ln) = M_z(x),$$

where $2L$ is the period of the domain structures, M_0 is the amplitude of the magnetization, n is the integer. Here we neglect the finite width of the domain walls (DW's) and assume that this width is much smaller than other length scales. We believe that the considered type of the S/F hybrids with the stripe-type domain structure could possibly be of interest as an object suitable for the manipulation of the 1D superconducting channels, formed in such a magnetic landscape, and by tuning the external magnetic field. Besides that this model can be quite relevant for the understanding of the peculiarities of the OP nucleation in real S/F hybrids with the F domains of the elongated shape,[15,16] when the shorter size of the domains is shown to determine mainly the phase boundary $T_c(H)$, see Ref. 16.

Both lateral $b_x(x,z)$ and transverse $b_z(x,z)$ components of the stray magnetic field outside the F film can be calculated analytically as follows (Ref. 19):

$$-b_x + ib_z = 4M_0\left[\ln\tan\frac{\pi w}{2L} - \ln\tan\frac{\pi(w-iD_F)}{2L}\right]. \quad (3)$$

Here $w=x+iz$, the planes $z=0$ and $z=D_F$ are the bottom and top surfaces of the F film. Typical spatial distributions $b_x(x,z)$ and $b_z(x,z)$ for such S/F hybrids are shown in Fig. 2.

The vector potential distribution $\mathbf{a}(\mathbf{r})$, corresponding to the inhomogeneous component $\mathbf{b}(\mathbf{r})$ of the magnetic field, can be written in the form of the even periodic function of the x coordinate, $\mathbf{a}(\mathbf{r})=a_y(x,z)\mathbf{y}_0$, see Eq. (A2). Choosing the gauge $\mathbf{A}=[a_y(x,z)+Hx]\mathbf{y}_0$, one can easily see that the Schrödinger-like Eq. (2) does not depend on a y coordinate, hence we can find the solution in the form $\Psi(\mathbf{r})=f_k(x,z)\exp(-iky)$, where the function $f_k(x,z)$ should be determined from the following 2D problem:

$$-\frac{\partial^2 f_k}{\partial x^2} - \frac{\partial^2 f_k}{\partial z^2} + V(x,z,k)f_k = \frac{1}{\xi^2(T)}f_k, \quad (4)$$

where

$$V(x,z,k) = \left[\frac{2\pi}{\Phi_0}a_y(x,z) + \frac{2\pi}{\Phi_0}Hx - k\right]^2. \quad (5)$$

We define the superconducting critical temperature T_c as the highest possible value $T_c=\max T_c(k)$, corresponding to the lowest "energy level" $E=D_F^2/\xi^2(T)$ of Eq. (4).

In order to complete the problem definition, one should add the boundary conditions. If the S film has an insulating interface at the top and bottom surfaces,

$$\left.\frac{\partial f_k}{\partial z}\right|_{z=D_F+h} = 0, \quad \left.\frac{\partial f_k}{\partial z}\right|_{z=D_F+h+D_s} = 0. \quad (6)$$

Now we discuss briefly the symmetry properties of Eqs. (4) and (5) to formulate the boundary conditions along the x axis. At $H=0$ the potential profile $V(x,z,k)$ becomes the even periodic function of the x coordinate. Note, that the double transformation $M_0\to -M_0$ and $x\to x-L$ does not change both $a_y(x,z)$ and $V(x,z,k)$, hence we have the same differential equations for $f_k(x,z)$ and $f_k(x-L,z)$ with the same eigenenergy $E_k\propto 1/\xi^2$. Since the ground state in the 1D case should be the nondegenerate function, so $f_k(x,z)=\alpha f_k(x-L,z)$. On the other hand, the wave function $f_k(x,z)$ in the presence of the periodic potential should satisfy the Bloch theorem:

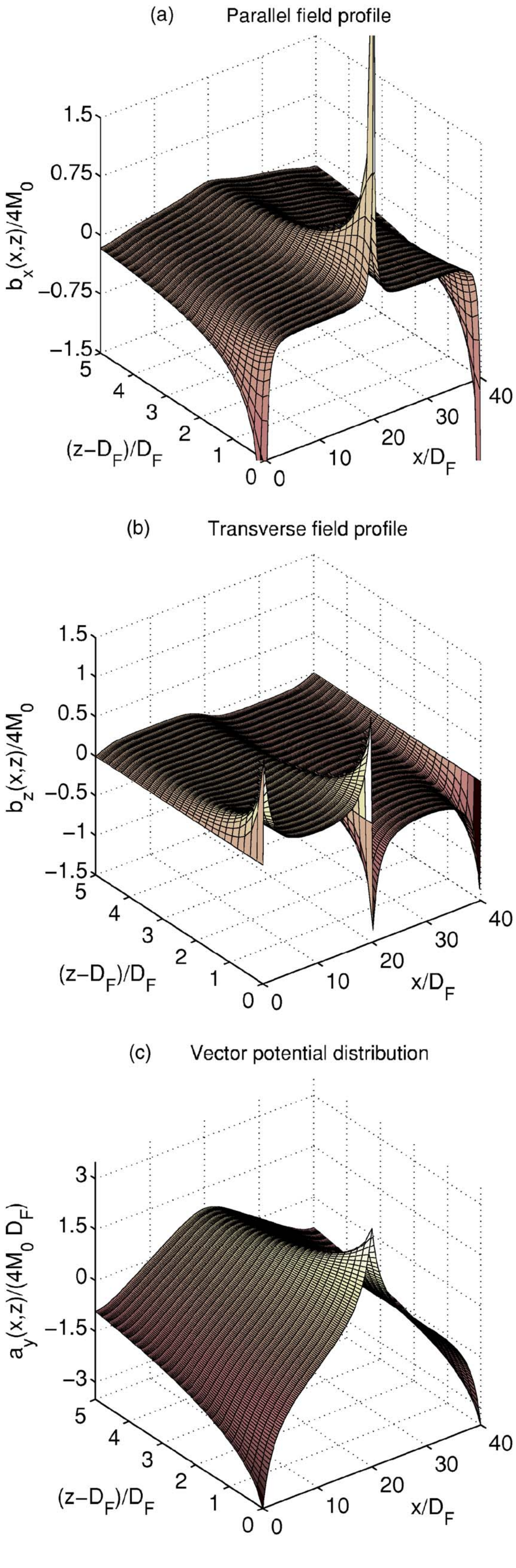

FIG. 2. (Color online) The parallel $b_x(x,z)$ (a) and transverse $b_z(x,z)$ (b) field distributions, induced by the one-dimensional domain structure (3) with the ratio $L/D_F=20$, as well as the corresponding vector potential profile $a_y(x,z)$ (c).

$f_k(x-2L,z)=f_k(x,z)e^{i2Lq}$. Noting that the ground state should correspond to the nodeless function, then $q=0$ and the Bloch theorem reduces to a simple periodicity condition: $f_k(x-2L,z)=f_k(x,z)$. Thus, we come to a conclusion, that $\alpha^2=1$ and the positive root ($\alpha=1$) should be chosen in order to avoid the sign inversion of $f_k(x,z)$. It allows one to solve Eq. (4) inside one half-period, $0\leqslant x\leqslant L$, with the boundary conditions

$$\left.\frac{\partial f_k}{\partial x}\right|_{x=0}=0, \quad \left.\frac{\partial f_k}{\partial x}\right|_{x=L}=0 \quad \text{for } H=0, \tag{7}$$

which are the logical consequences of the continuity of both $f_k(x,z)$ and $\partial f_k(x,z)/\partial x$, as well as the considered translation property $f_k(x)=f_k(x-L,z)$ and the evenness of the OP distribution, $f_k(x,z)=f_k(-x,z)$.

For $H\neq 0$ the translation symmetry along the x axis is broken and the Bloch theorem is no longer valid. Due to the parabolic increase of the effective potential $V(x,z,k)\propto(Hx)^2$ at $|x|\to\infty$, one can find the solution of Eq. (4) for the sample, infinite in the lateral direction, in the form of a localized nucleus

$$f_k|_{|x|\to\infty}=0 \quad \text{for } H\neq 0. \tag{8}$$

It is worth noting that here we do not take into account the edge OP nucleation. Let us introduce the typical lateral size $\mathcal{L}$ of the sample for some estimates. Despite the T_c value for the edge nucleus is higher than for other nuclei,[18] the area $S^{edge}\sim\xi\mathcal{L}$, which becomes superconducting due to the edge nucleation, seems to be much smaller than the area S^{bulk}, corresponding to the "bulk" nucleation (far from the edge). For instance, near the compensation field the OP nucleation takes place above the half of the domains, so $S^{bulk}\sim\mathcal{L}^2/2$ and we have $S^{edge}/S^{bulk}\sim\xi/\mathcal{L}\ll 1$ for large-area samples. Thus, the main contribution to the formation of the superconducting phase, which can be detected experimentally, should be attributed to the appearance of the nuclei far from the sample edge, so the use of the condition (8) appears to be quite reasonable.

III. RESULTS AND DISCUSSION

We start with the cases, when the thickness of the S film, D_s, is much smaller than other relevant length scales D_F and L (III A and III B). It allows us to neglect the field and the OP variations in the z direction, to omit the term $\partial^2 f_k/\partial z^2$ in Eq. (4), and to make the substitution $a_y(x,z)\to a_y(x,z^*)$, where the point $z^*=D_F+h+D_s/2$ corresponds to the middle line of the S film. As a result, the OP nucleation in infinitely thin S films is determined by the spatial profile of the perpendicular magnetic field only, while the effect of the parallel field can be neglected.

By decreasing the spacer distance h we can get the highest amplitude of the field modulation for the domain structure with given parameters M_0, L, and D_F. So the case $D_s\to 0$ and $h\to 0$ is of a particular interest as the ultimate case, when the influence of the modulated magnetic field on the OP nucleation should be the strongest (III A). By considering $h\neq 0$, we can effectively describe the continuous variation of the

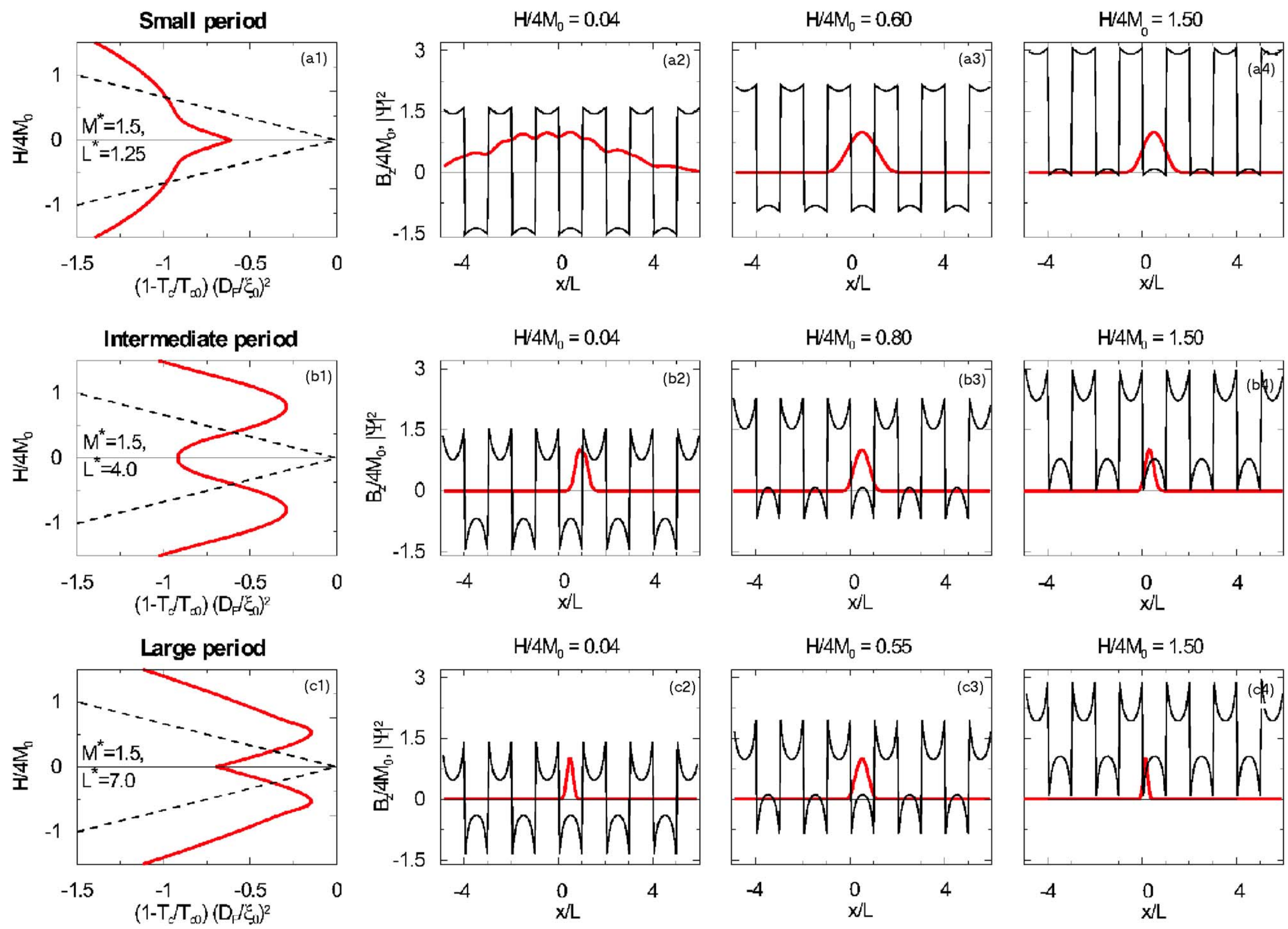

FIG. 3. (Color online) (a1), (b1), (c1) The phase transition lines $T_c(H)$ for $M^*=8\pi M_0 D_F^2/\Phi_0=1.5$ and for different periods $L^*=L/D_F$: $L^*=1.25$ (a1), $L^*=4.0$ (b1), $L^*=7.0$ (c1). Black dashed lines correspond to the $T_c(H)$ dependence in the absence of the nonuniform field. (a2)–(a4), (b2)–(b4), (c2)–(c4): The OP profiles $|\Psi(\mathbf{r})|^2=|f_k(x,z^*)|^2$ (a.u.) and the total magnetic field distributions $B_z(x,z^*)=b_z(x,z^*)+H$ for $M^*=1.5$ and $L^*=1.25,4.0,7.0$ (from the first row to the third one) in the presence of the external field H.

magnetic field near DW's, that allows us to understand the influence of the finite width DW's on the OP nucleation in real samples (III B).

Finally we present our results, related to the OP nucleation in the S/F hybrids with the S films of a finite thickness (III C), when the dependence of both components of the magnetic field on the z coordinate is crucial.

Note that the parameters of the ferromagnets with out-of-plane magnetization can be different: $L\sim 2$ μm, $D_F \simeq 90$ μm, $L/D_F\sim 0.02$ for the $BaFe_{12}O_{19}$ single crystals;[15] $L\sim 300$ nm, $D_F\simeq 20$ nm, $L/D_F\sim 15$ for the multilayer Co/Pd structures.[16] Since the spatial distribution of the magnetic field could be quite different for $L/D_F\ll 1$ and $L/D_F\gg 1$, we consider all these cases in detail.

A. Infinitely thin superconducting film near ferromagnetic film: $D_s\ll D_F$, L and $h/D_F\ll 1$

Depending on parameters M_0, L, and D_F, there are several OP nucleation regimes and several possible types of the phase boundary $T_c(H)$.

Small L case. For the S/F hybrids with relatively small periods of the field modulation $(L/D_F\ll 1)$ the OP distribution cannot always follow the rapid field's variation. There are practically uniform OP distribution at $H=0$ or a wide superconducting nucleus, spreading over several domains, at $|H|/4M_0\ll 1$ [Fig. 3(a2)]. The width of such nucleus ℓ is determined mainly by the external field H, similar to the nucleus' width in a uniform magnetic field: $\ell\sim L_H$, $L_H=\sqrt{\Phi_0/2\pi|H|}$ is the magnetic length. Based on this analogy, the critical temperature for such an S/F system is expected to decrease monotonically with increasing $|H|$: the smaller L, the closer dependence $T_c(H)$ to the classical linear form (1) is. By applying the external field we can shrink the nucleus' width and localize this nucleus within one half period above the domains with opposite magnetization [panels 3(a3) and 3(a4) in Fig. 3]. The interplay between two factors (both the external and periodic stray fields), which determines the resulting nucleus' width, leads in the sign change of the second derivative d^2T_c/dH^2. At high H values the width of the nucleus, positioned at the center of the magnetic domain [Fig. 3(a4)], is determined by the local field $B_{loc}\simeq|H|-\Delta H$, $\Delta H\simeq 2\pi M_0$, therefore we come to the shifted linear asymptotics

$$\frac{T_c}{T_{c0}} \simeq 1 - \frac{|H| - 2\pi M_0}{H_{c2}^0}, \quad |H|/4M_0 \gg 1. \tag{9}$$

Intermediate L case. For the S/F hybrids with the larger periods (the criterion will be presented below) the OP distribution becomes more sensitive to the presence of the inhomogeneous field, which leads to the appearance of the reentrant superconductivity. Even at $|H|/4M_0 \ll 1$ the OP distribution can possess well defined maxima near DW's [see Fig. 3(b2)], so this regime can be called "the domain wall superconductivity" (DWS). The typical phase boundary $T_c(H)$, corresponding to the DWS regime at $H=0$, is characterized by the presence of the pronounced reentrant superconductivity and the parabolic dependence T_c on H at low fields [Fig. 3(b1)]. A phase boundary of such type was predicted in Refs. 4 and 5 and later on observed experimentally.[15,16] Due to the field enhancement near DW's the superconducting nucleus is localized near DW's not only at $H \simeq 0$, but also at high fields [Fig. 3(b4)]. Based on the same reasons as above, one can conclude, that the asymptotical expression (9) for the dependence $T_c(H)$ in high-field limit should be also valid.

Large L case. For the S/F hybrids with the largest periods ($L/D_F \gg 1$) the magnetic field inside the magnetic domains is very inhomogeneous: there are deep minima in the $|b_z(x,z^*)|$ profile at the domains' centers (DC's), which are to be examined as the favorable positions for the OP nucleation. Thus, if the superconductivity could actually arise at the DC's at $H=0$, then the weak external field H should remove the equivalence of the domains: the superconducting nucleus will form in the center of domains with negative magnetization for $H>0$ and vice versa. The jumps of the superconducting nucleus between the regions with the different values of the local magnetic field at sign inversion of H should result in a type of phase boundary $T_c(H)$ with the singularity at $H=0$.

Indeed, the absolute value of the z component of the magnetic field at the DC's differs from the external field H and can be estimated as $B_{loc} \simeq 4\pi M_0 D_F/L - |H|$ for $L/D_F \gg 1$. Replacing $|H|$ by B_{loc} in expression (1), we immediately obtain

$$\frac{T_c}{T_{c0}} = 1 - \left(\frac{\xi_0}{D_F}\right)^2 \cdot \frac{\pi M^*}{L^*} + \frac{|H|}{H_{c2}^0}, \tag{10}$$

where $M^* = 8\pi M_0 D_F^2/\Phi_0$ and $L^* = L/D_F$ are the dimensionless magnetization and the half period, respectively. Thus, the regime of the domain center superconductivity (DCS) at $H/4M_0 \ll 1$ can be clearly identified by the appearance of the kink in the $T_c(H)$ dependence at $H=0$. Important to note, that at $L/D_F \gg 1$ the critical temperature T_c *increases* linearly with the same slope $dT_c/dH|_{H=0} = T_{c0}/H_{c2}^0$ as the T_c value decreases in an applied uniform magnetic field H [Fig. 3(c1)]. Recently, the authors of Ref. 5 also investigated the nucleation in the 1D periodical magnetic field, however, the considered cases of the meanderlike and sinelike field distributions do not have the minima in the center of domains, and this type of $T_c(H)$ dependence was not studied.

M_0-L diagram. We present the summary diagram M_0-L

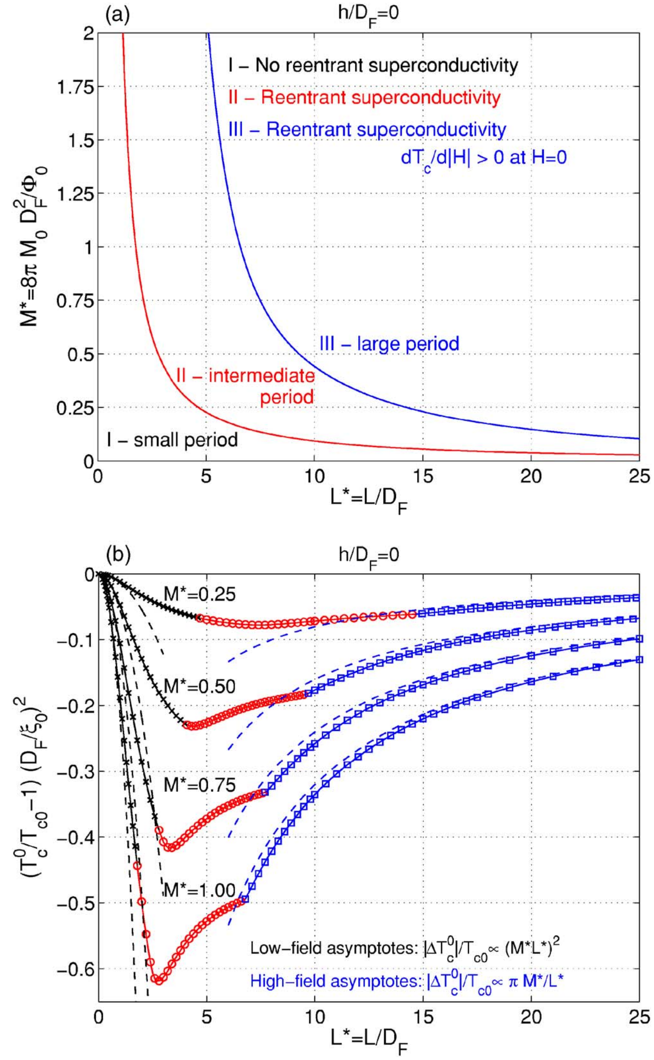

FIG. 4. (Color online) (a) The M_0-L diagram for $h/D_F=0$. In regions II and III the phase boundary $T_c(H)$ demonstrates the reentrant superconductivity. The slope $dT_c/d|H|$ at $H=0$ can be positive (III), zero (II) or negative (II, near the separating line I–II). The region I corresponds to the monotonic $T_c(H)$ dependence. (b) The suppression of the critical temperature $(\Delta T_c/T_{c0})(D_F/\xi_0)^2$ at $H=0$ as a function of the period $L^* = L/D_F$. Black (also marked by crosses), red (circles), and blue curves (squares) correspond to the regions I–III from the left panel, respectively. All dashed lines are the asymptotes, the analytical expressions are shown in the plot.

[Fig. 4(a)], which shows the regions I–III with the different regimes of the OP nucleation at $H=0$, which correspond to the different types of the phase transition lines $T_c(H)$. This diagram determines the threshold periods and separates the cases of the small (I), intermediate (II), and large periods (III), discussed above. Since these threshold periods depend on both M_0 and D_F (and also on the h parameter as it will be demonstrated later), it is rather difficult to formulate the reasonable conditions in a general way.

ΔT_c^0-L diagram. The different regimes of the superconducting nucleation can be easily seen on the dependence ΔT_c^0 vs L [Fig. 4(b)]. Here $\Delta T_c^0 = T_c^0 - T_{c0}$ is the suppression of the critical temperature at $H=0$ due to the influence of the inho-

mogeneous magnetic field only, i.e., $T_c^0 = T_c|_{H=0,M_0\neq 0}$.

For $L/D_F \ll 1$ (thick F films) the b_z profile has a steplike form, hence our problem is equivalent to the problem of the OP nucleation in the superconducting slab in the parallel uniform magnetic field[20] and $|\Delta T_c^0|/T_{c0} \propto (M^* L^*)^2 (\xi_0/D_F)^2$. For $L/D_F \gg 1$ (thin F films), when the DCS regime is realized, the suppression of T_c can be calculated using the universal formula (10), then $|\Delta T_c^0|/T_{c0} \simeq (\pi M^*/L^*)(\xi_0/D_F)^2$. These simple analytical expressions are in a nice agreement with the results of the numerical calculations at small and large L values [Fig. 4(b)]. We would like to note, that the position of the kink in the $\Delta T_c^0(L)$ dependence coincides exactly with the critical period, corresponding to the DWS-DCS transition.

B. Infinitely thin superconducting film above ferromagnetic film: $D_s \ll D_F$, L and $h/D_F = 1$

Due to the similarity between the magnetic field patterns for the S/F hybrids with the nonzero width of the DW's and for the considered systems at $h \neq 0$, this case of a nonzero h value is a key to qualitative understanding of the influence of the finite width of DW's on the nucleation process in the framework of our simple model.

M_0-L diagram. Due to the finite spacer distance the magnetic field inside the S film becomes much weaker and smoother than for $h=0$. As a result all lines, separating the different regions I–III on the M_0-L diagram, shift to the larger M_0 values [compare Fig. 4(a) and Fig. 5(a)]. We should note a significant enlargement of the intermediate region II, corresponding to the DWS regime at $H=0$. It means that the effective widening of the DW's suppress the OP nucleation at the DC's and stimulate the nucleation near the broadened DW's. Thus, it results in the formation of the phase transition line with zero slope dT_c/dH at $H=0$. We believe that these predictions should be valid for all hybrid S/F systems, since the zero width of DW's is clearly just a simplification.

ΔT_c^0-L diagram and the optimal period. For small L values and for $h \neq 0$ the magnetic field inside the S film is reduced to the sinelike profile with an extremely small amplitude of the order of $8M_0 e^{-\pi h/L}$, so that from the perturbation theory one can find the following universal estimate: $|\Delta T_c^0|/T_{c0} \propto (M^* L^*)^2 (\xi_0/D_F)^2 \ e^{-2\pi h/L}$. In the limit $L/D_F \gg 1$ (the DCS regime) the universal expression (10) is also valid, therefore $|\Delta T_c^0|/T_{c0} \simeq (\pi M^*/L^*)(\xi_0/D_F)^2$. The plateau in the $\Delta T_c^0(L)$ line for the intermediate L values corresponds to the OP nucleation near the broadened DW's (the DWS regime). All results are shown in Fig. 5(b).

We found out the existence of the optimal domain width L_{opt}, corresponding to the strongest influence of the nonuniform magnetic field on the critical temperature of the S film (the maximum in the $|\Delta T_c^0(L)|$ dependence). Probably, the domain structure with such parameters can be used in various applications as a candidate for the most effective interaction between the superconducting condensate and the nonuniform magnetic field. The optimal values L_{opt}^* can be determined approximately as $L_{opt}^*(M^*, h^*) \simeq (1.1 h^* \sqrt{M^*} + 1) L_{opt}^*(M^*, 0)$ where $h^* = h/D_F$, $L_{opt}^*(M^*, 0) \simeq A(M^*)^n$; for

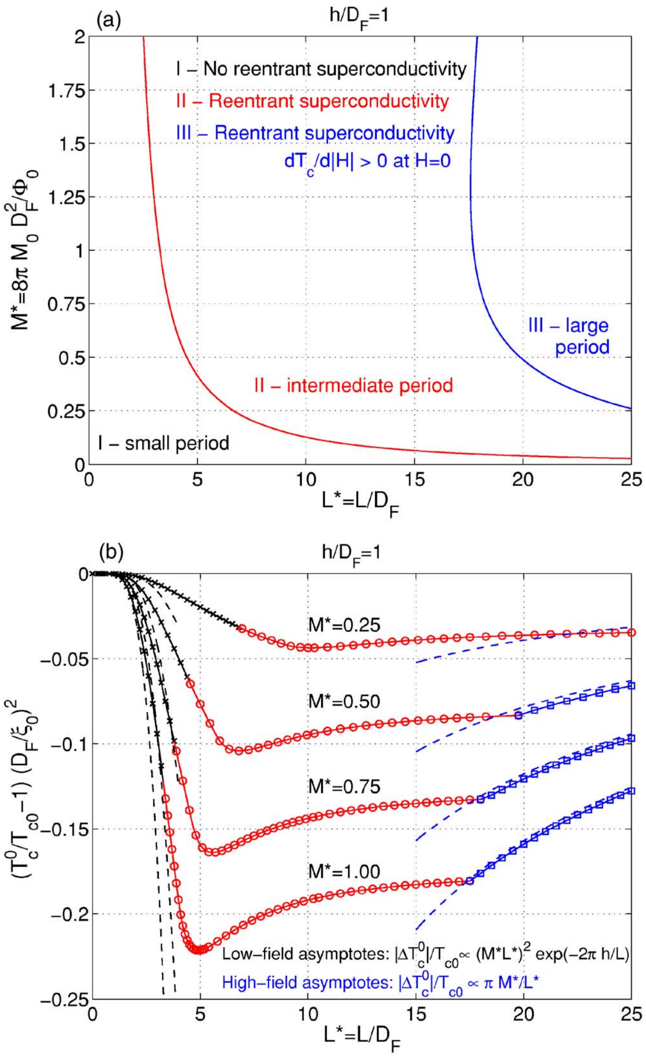

FIG. 5. (Color online) (a) The M_0-L diagram for $h/D_F = 1$. In regions II and III the phase boundary $T_c(H)$ demonstrates the reentrant superconductivity. The slope $dT_c/d|H|$ at $H=0$ can be positive (III), zero (II) or negative (II, near the separating line I–II). The region I corresponds to the monotonic $T_c(H)$ dependence. (b) The suppression of the critical temperature $(\Delta T_c/T_{c0})(D_F/\xi_0)^2$ at $H=0$ as a function of the period $L^* = L/D_F$. Black (also marked by crosses), red (circles), and blue curves (squares) correspond to the regions I–III from the left panel, respectively. All dashed lines are the asymptotes, the analytical expressions are shown in the plot.

$M^* \lesssim 0.5$ $A \simeq 1.55$, $n \simeq -0.75$; for $0.5 \lesssim M^* \lesssim 1.0$ $A \simeq 1.80$, $n \simeq -0.60$; for $M^* \gtrsim 1.0$ $A \simeq 1.65$, $n \simeq -0.42$. The disarrangement between the exact numerical results and this scaling formula is no more than 10%.

C. Thick superconducting film near ferromagnetic film: $D_s \neq 0$ and $h/D_F \ll 1$

The problem of the nucleation of superconductivity in the S films of a finite thickness is more complicated and it requires a lot more computing work even for our simple

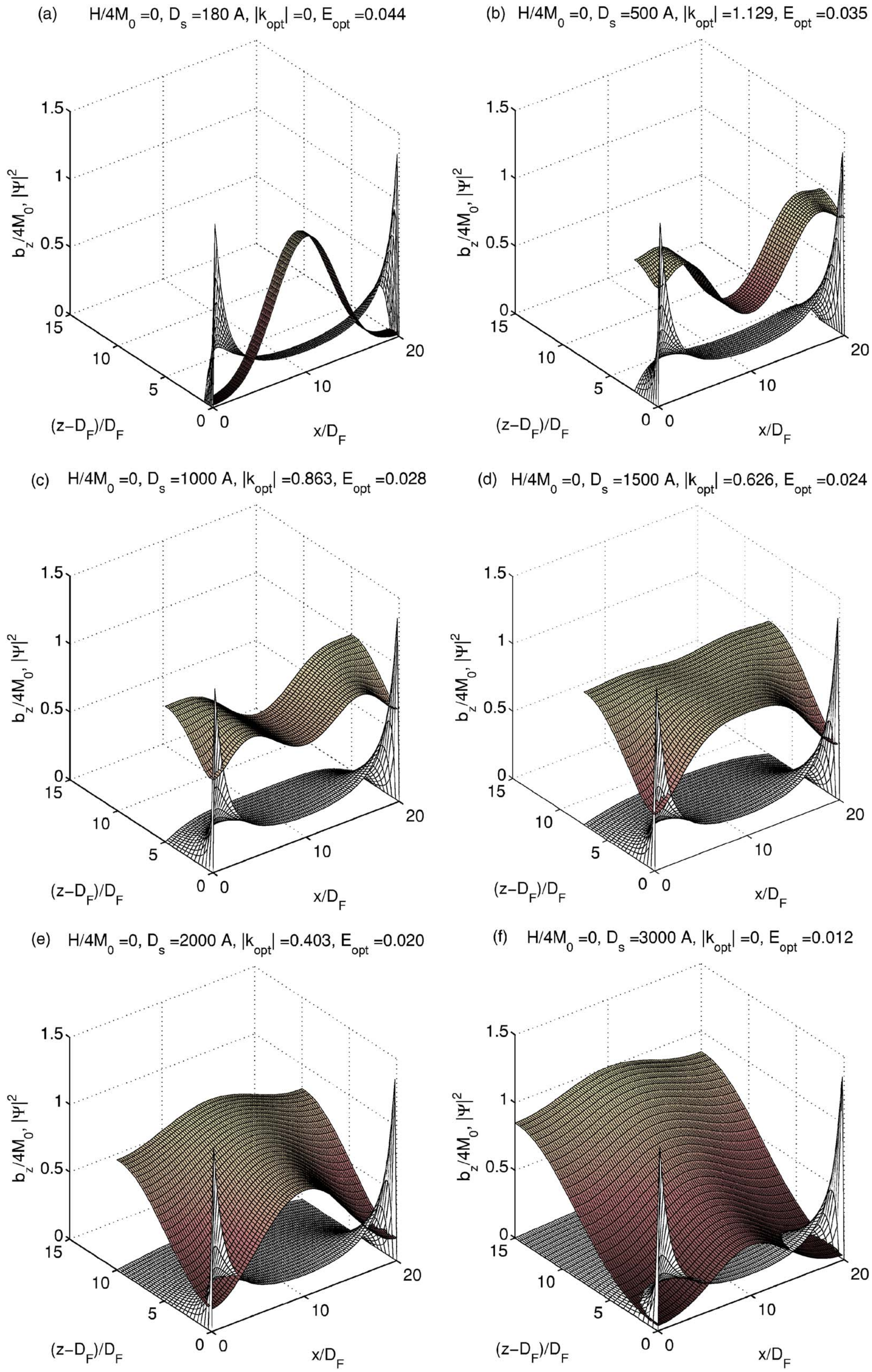

FIG. 6. (Color online) The OP distributions $|\Psi(x,z)|^2$ (a.u.) at $H=0$ and the transverse field profiles $b_z(x,z)$ for the S/F hybrids with the S films of the different thicknesses: $D_s=180$ Å (a), $D_s=500$ Å (b), $D_s=1000$ Å (c), $D_s=1500$ Å (d), $D_s=2000$ Å (e), $D_s=3000$ Å (f). Other parameters of the system are $M_0=500$ Oe, $L=4000$ Å, $D_F=200$ Å, $h=5$ Å.

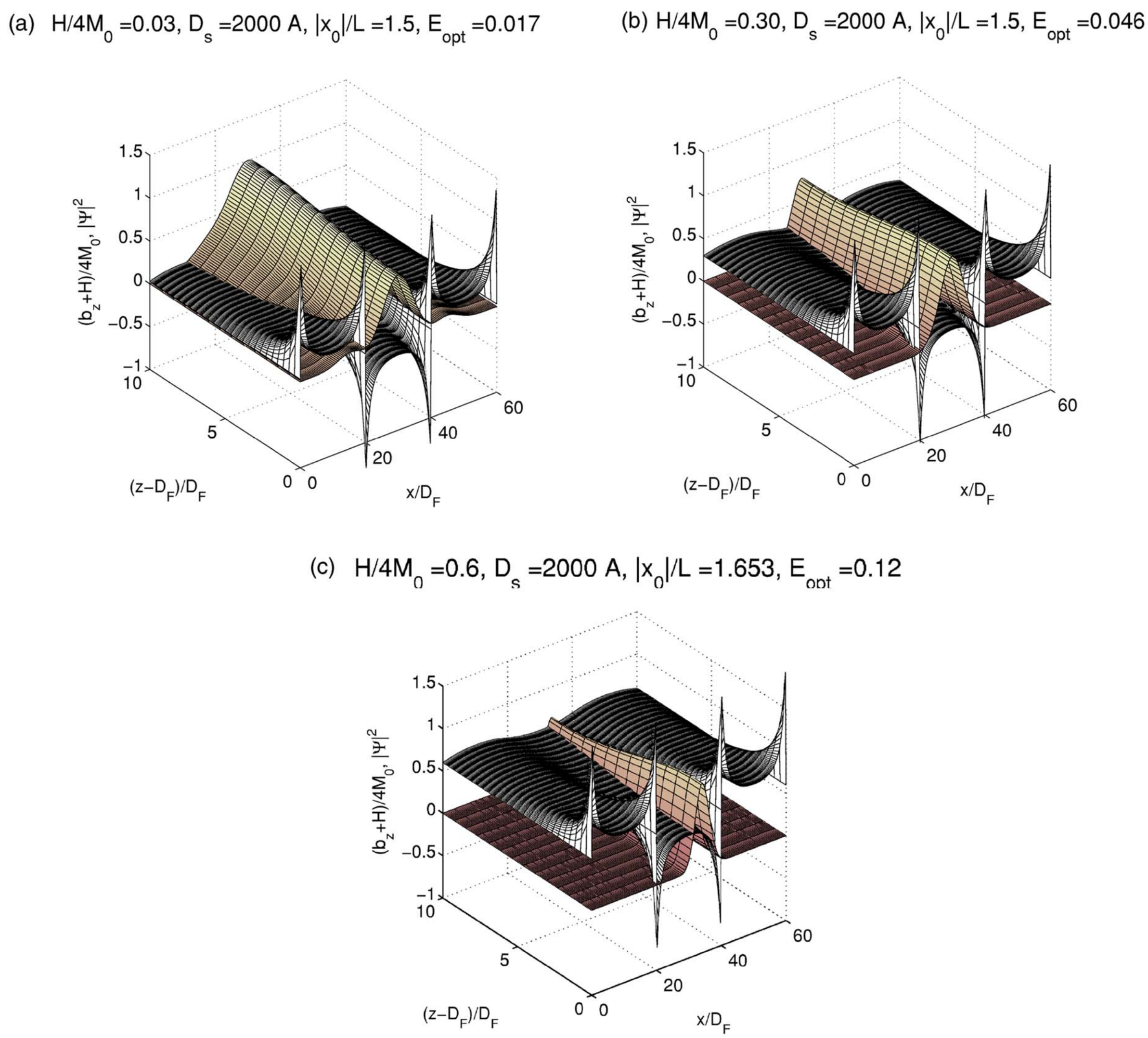

FIG. 7. (Color online) The OP distributions $|\Psi(x,z)|^2$ (a.u.) and the transverse field profiles $B_z(x,z)=b_z(x,z)+H$ in thick S/F hybrids with D_s=2000 Å in the presence of the external field H: $H/4M_0$=0.03 (a), $H/4M_0$=0.30 (b), $H/4M_0$=0.60 (c). Other parameters are the following: M_0=500 Oe, L=4000 Å, D_F=200 Å, h=5 Å.

model. Due to that we considered only certain S/F hybrids with the parameters close to the experimental ones:[16] $M_0 \simeq 500$ Oe, L=4000 Å, D_F=200 Å, h=5 Å, which correspond to the reduced values M^*=0.25, L^*=20, $h^* \ll 1$. Such choices of the parameters provides us with the realization of the DCS regime for infinitely thin films at H=0 [with the kink at the $T_c(H)$ line]. Since this DCS regime is the most sensitive to the variation of the parameters, we hope to observe all stages in the transformation of the dependence $T_c(H)$ at increasing the D_s value. Based on the considered cases, we will draw some general conclusions about the effect of the S film thickness on the OP nucleation.

OP nucleation at H=0. Due to the translation symmetry, the problem (4) can be solved only at one half period. The exponential decay of both components of the magnetic field, $b_x, b_z \propto 4M_0 e^{-\pi z/L}$, modifies the shape of the nucleus in the S film of a finite thickness and raises the critical temperature of the appearance of such nucleus. In particular, the increase of the D_s value stimulates the subsequent transformation of the quasiuniform OP distribution along the z-coordinate for $D_s \ll L$ to the one, which has the maximum of the OP at the distant (top) surface of the S film [panels 6(a)–6(f) in Fig. 6]. It is worth noting the existence of some threshold thickness D_s^{cr}, which separates the DCS regime for smaller D_s values from the DWS regime at $D_s > D_s^{cr}$. For our choice of parameters $D_s^{cr} \simeq 200$ Å, i.e., it is of the order of D_F.

OP nucleation at $H \neq 0$. The direct numerical solution of Eq. (4) for $D_s \neq 0$ indicates that the applied external field makes the OP variations in the transverse direction less preferable than in the lateral direction (see Fig. 7), especially for the H values of the order of the compensation field or higher. It can be explained by noting the certain nonequivalence between the x and z axes, arising from the symmetry of the considered problem and from the linear growth of the vector potential in the x direction only, $A_y = a_y(x,z) + Hx$. For instance, the OP distribution even for D_s=1000 Å becomes practically uniform over the sample thickness, starting from $|H|/4M_0 > 0.05$.

Perturbation theory. Assuming that the superconducting nucleus in the relatively thin S film (in comparison with the typical length scales L and D_F) is almost uniform over the thickness, we can reduce the 2D LGL Eq. (4) to a simpler version. One can expand the complicated expression for the vector potential $A_y(x,z)$ inside the S film into Taylor series:

$$A_y(x,z) = a_y(x,z^*) + Hx + \left(\frac{\partial a_y}{\partial z}\right)_{x,z^*}(z-z^*) + \frac{1}{2}\left(\frac{\partial^2 a_y}{\partial z^2}\right)_{x,z^*}(z-z^*)^2 + \cdots , \qquad (11)$$

and choose the point z^* at the middle line of the S film. After substituting expression (11) in Eq. (4), and integrating Eq. (4) over the variable $z' = z - z^*$ in the interval from $-D_s/2$ to $D_s/2$, we get the following 1D equation for the averaged OP distribution $\bar{f}_k(x)$,

$$-\frac{\partial^2 \bar{f}_k}{\partial x^2} + V_0(x)\bar{f}_k + V_1(x)\bar{f}_k = \frac{1}{\xi^2(T)}\bar{f}_k, \qquad (12)$$

where the inhomogeneity of the magnetic field along the z axis can be taken into account by means of the additional potential well $V_1(x)$

$$V_0(x) = \left[\frac{2\pi}{\Phi_0}a_y(x,z^*) + \frac{2\pi}{\Phi_0}Hx - k\right]^2,$$

$$V_1(x) = \frac{(2\pi)^2 D_s^2 b_x^2(x,z^*)}{12\Phi_0^2} - \frac{2\pi D_s^2 b_z'(x,z^*)}{12\Phi_0} \times \left[\frac{2\pi}{\Phi_0}a_y(x,z^*) + \frac{2\pi}{\Phi_0}Hx - k\right].$$

Phase boundaries $T_c(H)$ for $D_s \neq 0$. The $T_c(H)$ dependences for the S/F structures with the different thicknesses $D_s = 0 \cdots 2000$ Å, obtained by the numerical solution of the 2D Eq. (4) and the perturbed 1D Eq. (12), are shown in Fig. 8 as symbols and lines, correspondingly. Examining these results, we can conclude the following

(i) The DCS nucleating regime at $H=0$ is still realized for rather thin S films, however, the increase the D_s value causes the transition from the DCS regime to the DWS and results in the appearance of zero slope in the $T_c(H)$ dependence at $H=0$ even for the S/F hybrids with the relatively large periods.

(ii) A nice agreement between both 1D and 2D solutions in the considered range of the D_s values confirms our expectations about the quasiuniform nature of the OP distribution in the S samples of a finite thickness at $H \neq 0$. The developed approach (12) can therefore be applied for the determination of the phase transition line $T_c(H)$ even for rather thick S samples without solving the complicated 2D problem.

(iii) At rather weak external fields the approximate method, based on the 1D equation (12), should be less adequate as the the thickness of the S film increases and becomes comparable or larger than L.

(iv) The dependence $T_c(H)$ for the rather thick S films in the high-field limit tends to the phase transition line described by Eq. (1), which is inherent for the large-area S samples in a uniform magnetic field. It follows from the

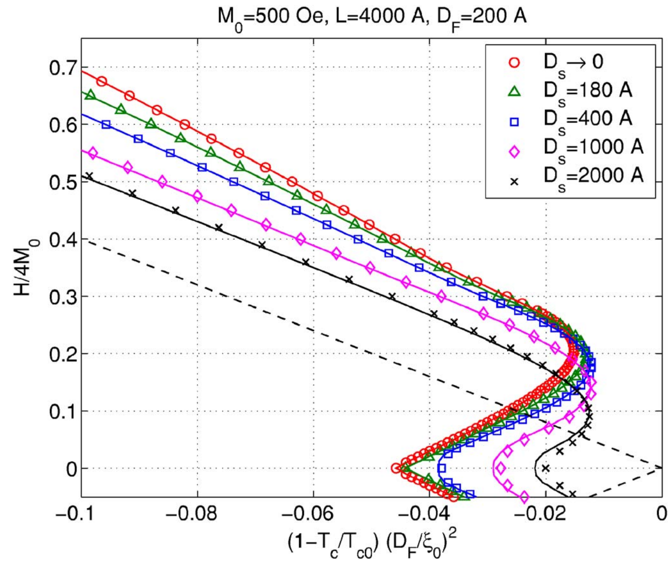

FIG. 8. (Color online) The transformation of the phase transition line $T_c(H)$ with increasing the thickness D_s of the superconducting film: $D_s \to 0$ (red ○), $D_s=180$ Å (green △), $D_s=400$ Å (blue □), $D_s=1000$ Å (magenta ◇), and $D_s=2000$ Å (black ×). The symbols represent the results of the numerical solution of the 2D equation (4), while the solid lines are the solutions of the perturbed 1D equation (12). The dashed line is the reference dependence (1) for $M_0=0$.

effective averaging of the inhomogeneous magnetic field by the quasiuniform nucleus over the sample thickness, which substantially weakens the effect, arising from the field modulation in the lateral direction.

IV. CONCLUSION

In this paper we theoretically investigated the OP nucleation in the planar S/F bilayer hybrids, taking into account specific distribution of the stray magnetic fields in such systems. Our main results are the following:

(i) The analytical expression for the vector potential $a_y(x,z)$ has been obtained, which is suitable for the calculation of both components of the stray field $b_x(x,z)$ and $b_z(x,z)$ at an arbitrary distance above the F film.

(ii) We presented the systematical treatment of the influence of the inhomogeneity of the magnetic field both in the lateral and transverse directions on the OP distribution $|\Psi(x,z)|^2$ and on the $T_c(H)$ dependence. We demonstrated, that the lateral inhomogeneity is the cause of the exotic phase transition line $T_c(H)$, while the field variations over the thickness of the S sample only weakens the influence of the nonuniform magnetic field on the OP nucleation.

(iii) A type of phase boundary $T_c(H)$ with the positive slope $dT_c/d|H|$ at $H=0$ has been predicted. Such a type of the dependence $T_c(H)$ unambiguously corresponds to the OP nucleation in the centers of the magnetic domains at $H=0$, which should be realized in the S/F hybrids for $L/D_F \gg 1$ and $D_s \lesssim D_F$. We studied the competition between the OP nucleation above the centers of magnetic domains and above the domain walls by varying the parameters of the S/F structures.

(iv) An approximate method for the calculation of the $T_c(H)$ line for the S/F hybrids of a finite thickness has been developed, which provides quite good accuracy. The main idea is the reduction of the 2D linearized Ginzburg-Landau equation to the perturbed 1D equation for the OP distribution, averaged over the S film thickness.

(v) We predicted the decrease in the difference between the high-field part of the phase transition line $T_c(H)|_{M_0\neq 0}$ and the reference dependence $T_c(H)|_{M_0=0}$ at increasing the thickness of the S film.

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APPENDIX: THE VECTOR POTENTIAL DISTRIBUTION

The easiest way to find the vector potential, corresponding to expression (3), is as follows. Taking into account the symmetry of the problem, we assume $\mathbf{a}=a_y(x,z)\mathbf{y}_0$. The magnetic field $\mathbf{b}$ outside the ferromagnetic film can be described both by the scalar potential $\mathbf{b}=-\nabla\varphi$ and by the vector potential $\mathbf{b}=\text{rot}\,\mathbf{a}$ due to the Maxwell's equations: rot $\mathbf{b}=0$ and div $\mathbf{b}=0$. We can introduce the complex potential $U(x,z)=\varphi(x,z)+ia_y(x,z)$, which is the analytical function of the complex argument $w=x+iz$. Since $dU/dw=-b_x+ib_z$, one can get

$$\frac{dU}{dw}=4M_0\left[\ln\tan\frac{\pi w}{2L}-\ln\tan\frac{\pi(w-iD_F)}{2L}\right]. \tag{A1}$$

Integrating Eq. (A1) over w, one can obtain

$$\begin{aligned}U(w)=&-\frac{4M_0Li}{\pi}\cdot\left[\ln\left(\tan\frac{\pi w}{2L}\right)\cdot\ln\frac{1+i\tan\pi w/2L}{1-i\tan\pi w/2L}\right.\\&\left.+\,\text{dilog}\left(1+i\tan\frac{\pi w}{2L}\right)-\text{dilog}\left(1-i\tan\frac{\pi w}{2L}\right)\right]\\&+\frac{4M_0Li}{\pi}\cdot\left[\ln\left(\tan\frac{\pi w'}{2L}\right)\cdot\ln\frac{1+i\tan\pi w'/2L}{1-i\tan\pi w'/2L}\right.\\&\left.+\,\text{dilog}\left(1+i\tan\frac{\pi w'}{2L}\right)-\text{dilog}\left(1-i\tan\frac{\pi w'}{2L}\right)\right],\end{aligned}$$

where $w'=w-iD_F$ and the dilogarithmic function is determined as follows (Ref. 21):

$$\text{dilog}(t)=\int_1^t\frac{\ln\tau}{1-\tau}d\tau,\qquad\frac{d}{dt}\text{dilog}(t)=\frac{\ln t}{1-t}.$$

Thus, the vector potential $a_y(x,z)$, which is necessary for the solving of the Eq. (4), is just the imaginary part of the complex potential $U(w)$:

$$a_y(x,z)=\text{Im}\,U(x+iz). \tag{A2}$$

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