



ELSEVIER

Physica C 332 (2000) 417–421

PHYSICA C

www.elsevier.nl/locate/physc

Nucleation of superconductivity in a mesoscopic loop of finite width

Vital Bruyndoncx^{*}, Lieve Van Look, Victor V. Moshchalkov

Laboratorium voor Vaste-Stoffysica en Magnetisme, Katholieke Universiteit Leuven, Celestijnenlaan 200 D, B-3001 Leuven, Belgium

Abstract

The normal/superconducting phase boundary T_c has been calculated for mesoscopic loops as a function of an applied perpendicular magnetic field H . While for thin-wire loops and filled disks, the $T_c(H)$ curves are well known, the intermediate case, namely, mesoscopic loops of finite wire width, have been studied much less. The linearized first Ginzburg–Landau (GL) equation is solved with the proper normal/vacuum boundary conditions both at the internal and at the external loops radii. For thin-wire loops, the $T_c(H)$ oscillations are perfectly periodic, and the $T_c(H)$ background is parabolic (this is the usual Little–Parks effect). For loops of thicker wire width, there is a crossover magnetic field above which $T_c(H)$ becomes quasi-linear, with the period identical to the $T_c(H)$ of a filled disk (i.e., pseudoperiodic oscillations). This dimensional transition is similar to the 2D–3D transition for thin films in a parallel field, where vortices start penetrating the material as soon as the film thickness exceeds the temperature-dependent coherence length by a factor 1.8. For the presently studied loops, the crossover point is controlled by a similar condition. In the high-field ‘3D’ regime, a giant vortex state establishes, where only a surface superconducting sheath near the sample’s outer radius is present. © 2000 Elsevier Science B.V. All rights reserved.

PACS: 74.60.Ec; 74.25.Dw; 73.23.-b; 74.20.De; 74.76.-w

Keywords: Ginzburg–Landau theory; Phase diagram; Coherence length; Vortex; Nucleation

1. Introduction

The nucleation of superconductivity in mesoscopic samples has received a renewed interest after the development of nanofabrication techniques, like

electron beam lithography. A superconductor is in the mesoscopic regime when the sample size is comparable to the superconducting coherence length $\xi(T)$. In the framework of the Ginzburg–Landau (GL) theory, $\xi(T)$ sets the length scale for spatial variations of the modulus of the superconducting order parameter $|\Psi|$. The pioneering work on mesoscopic superconductors was carried out already in 1962 by Little and Parks [1,2], who measured the shift of the critical temperature $T_c(H)$ of a (multiply-connected) thin-walled Sn microcylinder (a thin-wire “loop”) in an axial magnetic field H .

^{*} Corresponding author.

E-mail address: vital.bruyndoncx@fys.kuleuven.ac.be (V. Bruyndoncx).

The $T_c(H)$ phase boundary showed a periodic component, with the magnetic period corresponding to the penetration of a superconducting flux quantum $\Phi_0 = h/2e$.

A few years later, Saint-James calculated the $T_c(H)$ of a singly connected cylinder [3] (a mesoscopic ‘‘disk’’). Taking into account the analogy with the situation of a semi-infinite superconducting slab in contact with vacuum [4], the critical field was called $H_{c3}(T)$ in this case, since superconductivity nucleates initially near the sample interface. In the present paper, we will use the notation $H_{c3}^*(T)$, for the nucleation magnetic field.

The $T_c(H)$ phase boundary (or $H_{c3}^*(T)$) of the disk shows, like the usual Little–Parks effect in a multiply connected sample (loop), an oscillatory behaviour. When moving along $T_c(H)$, superconductivity concentrates more and more near the sample interface as H grows. A giant vortex state is formed: a ‘‘normal’’ core carries L flux quanta, and the ‘effective’ loop radius increases, resulting in a decrease of the magnetic oscillation period. An experimental verification of these predictions was carried out later on by Buisson et al. [5] and by Moshchalkov et al. [6].

In the early paper of Saint-James and de Gennes [4], $H_{c3}^*(T)$ has also been calculated for a film exposed to a parallel magnetic field, where surface superconductivity can grow along two superconductor/vacuum interfaces. For low magnetic fields, the two surface superconducting sheaths overlap, and, as a result, T_c vs. H becomes parabolic, which is a characteristic for 2D behaviour. When increasing the field, a crossover to a linear $T_c(H)$ dependence (3D) occurs at $t \approx 2\xi(T)$, with t as the film thickness. Shortly after, it was shown that vortices start to nucleate in the film at this dimensional crossover point ($t = 1.8\xi(T)$) [7,8].

The goal of the present paper is to study the phase boundary $T_c(H)$ of loops made of finite width wires. In a Type-II material, superconductivity is expected to be enhanced, with respect to the bulk upper critical field H_{c2} : $H_{c3}^*(T) > H_{c2}(T)$, both at the external and the internal sample surfaces. As for a film in a parallel field, a 2D–3D dimensional crossover can be anticipated, since the loops may be simply considered as a film, which is bent, such that its ends are joined together. We calculate, for the first time,

the phase boundary $T_c(H)$ as the ground state solution of the linearized first GL equation with two superconductor/vacuum interfaces. Calculations for a similar geometry, in the framework of the full nonlinear GL equations, were very recently carried out by Baelus et al. [9].

2. The linearized GL equation

The linearized first GL equation to be solved in order to find $T_c(H)$ is:

$$\frac{1}{2m^*} (-i\hbar\vec{\nabla} - 2e\vec{A})^2 \Psi = -|\alpha| \Psi. \quad (1)$$

This equation is formally identical to the Schrödinger equation for a particle with a charge $2e$ in a magnetic field. At the onset of superconductivity, the nonlinear GL term can be omitted and the z dependence disappears from the equations and, therefore, an infinitely long cylinder and a disk have identical $T_c(H)$ boundaries. It is further assumed that $\mu_0 \vec{H} = \text{rot } \vec{A}$, with H as the applied magnetic field. The eigenenergies $|\alpha|$ can be written as:

$$|\alpha| = \frac{\hbar^2}{2m^*\xi^2(T)} = \frac{\hbar^2}{2m^*\xi^2(0)} \left(1 - \frac{T}{T_{c0}}\right), \quad (2)$$

where T_{c0} is the critical temperature in zero magnetic field. From the energy eigenvalues of Eq. (1), the lowest Landau level $|\alpha_{\text{LLL}}(H)|$ is directly related to the highest possible temperature $T_c(H)$, for which superconductivity can exist.

For the loop geometries, we choose the polar coordinate system (r, φ) and the gauge $\vec{A} = (\mu_0 Hr/2)\vec{e}_\varphi$, where \vec{e}_φ is the tangential unit vector. The exact solution of the Hamiltonian (Eq. (1)) in cylindrical coordinates takes the following form [10–12]:

$$\Psi(\Phi, \varphi) = e^{-iL\varphi} \left(\frac{\Phi}{\Phi_0}\right)^{L/2} \exp\left(-\frac{\Phi}{2\Phi_0}\right) \times K(-n, L+1, \Phi/\Phi_0), \quad (3)$$

$$K(a, c, y) = c_1 M(a, c, y) + c_2 U(a, c, y).$$

Here, $\Phi = \mu_0 H \pi r^2$ is the applied magnetic flux through a circle of radius r . The number n determines the energy eigenvalue. Most generally, the function $K(a, c, y)$ can be any linear combination of

the two confluent hypergeometric functions (or Kummer functions), $M(a,c,y)$ and $U(a,c,y)$ [13], but the sample topology puts a constraint on c_1 , c_2 , and n , via the Neumann boundary condition:

$$(i\hbar\vec{\nabla} - 2e\vec{A})\Psi|_{\perp,b} = 0, \tag{4}$$

which the solutions Ψ of Eq. (1) have to fulfill at the sample interfaces b .

The eigenenergies of Eq. (1) can be written in the form:

$$\begin{aligned} \frac{r_o^2}{\xi^2(T_c)} &= \frac{r_o^2}{\xi^2(0)} \left(1 - \frac{T_c(H)}{T_{c0}} \right) = 4 \left(n + \frac{1}{2} \right) \frac{\Phi}{\Phi_0} \\ &= \epsilon(H_{c3}^*) \frac{\Phi}{\Phi_0}, \end{aligned} \tag{5}$$

where $\Phi = \mu_0 H \pi r_o^2$ is arbitrarily defined. The integer number L is the phase winding, or fluxoid quantum number. The parameter n depends on L and is not necessarily an integer number, as we shall see further on.

The bulk Landau levels are obtained when substituting $n = 0, 1, 2, \dots$ in Eq. (5), meaning that the lowest level $n = 0$ corresponds to the bulk upper critical field $\mu_0 H_{c2}(T) = \Phi_0 / (2\pi \xi^2(T))$.

For a disk geometry [3,5], we have to take $c_2 = 0$ in Eq. (3) in order to avoid the divergency of $U(a,c,y \rightarrow 0) = \infty$ at the origin. Selecting the lowest Landau level at each value Φ , one ends up with a cusp-like $T_c(H)$ phase boundary [3], which is composed of values $n < 0$ in Eq. (5), thus, leading to $H_{c3}^*(T) > H_{c2}(T)$. A similar calculation was performed for a single circular microhole in a plane film (“antidot”) [10,11], where $c_1 = 0$ in Eq. (3), since $M(a,c,y \rightarrow \infty) = \infty$. Here as well, the lowest Landau level consists of solutions with $n < 0$. At each cusp in $T_c(\Phi)$, the system makes a transition $L \rightarrow L \pm 1$, i.e., a flux quantum enters or is removed from the sample.

The loops we are currently studying have two superconducting/vacuum interfaces, one at the outer radius r_o , and one at the inner radius r_i . Consequently, the boundary condition (Eq. (4)) has to be fulfilled at both r_o and r_i . As a result, we have a system of two equations and two variables n and c_2

($c_1 = 1$ is chosen), which we solved for different values of r_i/r_o .

3. Results

Fig. 1 shows the Landau level scheme (dashed lines) calculated from Eqs. (3)–(5), for a loop with $r_i/r_o = 0.5$. The applied magnetic flux $\Phi = \mu_0 H \pi r_o^2$ is defined with respect to the outer sample area. The $T_c(H)$ boundary is composed of Ψ solutions with a different phase winding number L and is drawn as a solid cups-like line in Fig. 1. At $\Phi \approx 0$, the state with $L = 0$ is formed at $T_c(\Phi)$ and one by one, consecutive flux quanta L enter the loop as the magnetic field increases. For low magnetic flux, the background depression of T_c is parabolic, whereas at higher flux, $T_c(\Phi)$ becomes quasi-linear, like in the case of a filled disk. The crossover point from parabolic to quasi-linear appears at about $\Phi \approx 14\Phi_0$.

The solid and dotted straight lines in Fig. 1 are the bulk upper critical field $H_{c2}(T)$ and the surface critical field $H_{c3}(T)$ for a semi-infinite slab, respectively. In these units, the slopes of the curves (see Eq. (5)) are $\epsilon = 2$ for H_{c2} (substitute $n = 0$ in Eq. (5)) and $\epsilon = 2/1.69$ for H_{c3} . The ratio $\eta = \epsilon(H_{c2})/\epsilon(H_{c3}) = 1.69$ corresponds then to the en-

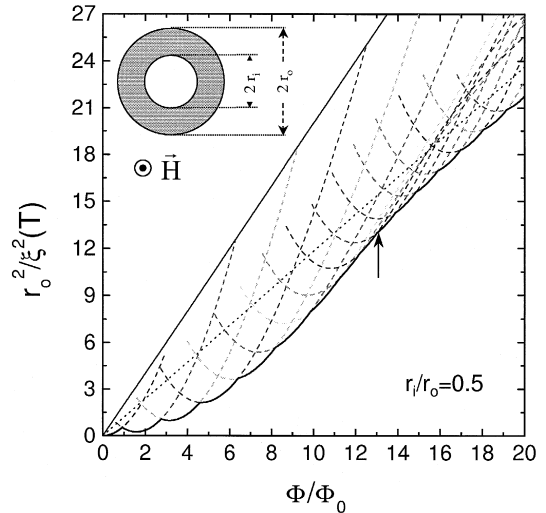


Fig. 1. Calculated energy level scheme (dashed lines) for a superconducting loop with the ratio of inner to outer radius $r_i/r_o = 0.5$. The solid and dotted lines correspond to $H_{c2}(T)$ and $H_{c3}(T)$, respectively.

hancement factor $H_{c3}(T)/H_{c2}(T)$ at a constant temperature. For the loops we are studying here, $\eta = \epsilon(H_{c2})/\epsilon(H_{c3}^*)$ is varying as a function of the magnetic field.

The energy levels below the H_{c2} line (solid straight line in Fig. 1) could be found by fixing a certain L , and finding the real numbers $n < 0$ numerically after inserting the general solution (Eq. (3)) into the boundary condition (Eq. (4)). Note that the lowest Landau level always has a lower energy $|\alpha(\Phi)|$ than a semi-infinite superconducting slab, which implies $H_{c3}^*(T) > H_{c3}(T) = 1.69H_{c2}(T)$.

As mentioned earlier, in a thin film of thickness t in a parallel field H , a dimensional crossover is found at $t = 1.84\xi(T)$. For low fields (high ξ), $T_c(H)$ is parabolic (2D), and for higher fields, vortices start penetrating the film and consequently, $T_c(H)$ becomes linear (3D) [7,8]. In Fig. 1, the small arrow indicates the point on the phase diagram $T_c(\Phi)$ where $w = 1.84\xi(T)$. For the loops as well, the dimensional transition shows up approximately at this point, although the vortices are not penetrating the sample area in the 3D regime. Instead, the middle loop opening contains a coreless ‘giant vortex’ with an integer number of flux quanta $L\Phi_0$.

In order to compare the flux periodicity of $T_c(\Phi)$, we have plotted, in Fig. 2, the lowest energy levels of Fig. 1 as $\eta^{-1} = \epsilon(H_{c3}^*)/\epsilon(H_{c2})$, for loops with different r_i/r_o . In this representation, the dotted horizontal line at $\eta^{-1} = 0.59$ corresponds to the surface critical field line $H_{c3}(T)$. The nucleation field of a disk $H_{c3}^*(T) > 1.69H_{c2}(T)$, and for a circular microhole in an infinite film (‘antidot’) [10,11], $H_{c3}^*(T) < 1.69H_{c2}(T)$. As Φ grows (the radius goes to infinity), the $H_{c3}^*(T)$ of both the disk and the antidot approaches the $H_{c3}(T)$ line.

For all the loops we study here, the presence of the outer sample interface automatically implies that $H_{c3}^*(T) > H_{c3}(T)$ is enhanced ($\eta > 1.69$), with respect to the case of a flat superconductor–vacuum interface. For loops with a small r_i/r_o , the $T_c(\Phi)$ boundary very rapidly collapses with the $T_c(\Phi)$ of the dot (η becomes the same). The presence of the opening in the sample is not relevant for the giant vortex formation in the high-flux 3D regime. On the contrary, in the low-flux regime, the surface sheaths along the two interfaces overlap, giving rise to a different periodicity of $T_c(\Phi)$ and to a parabolic

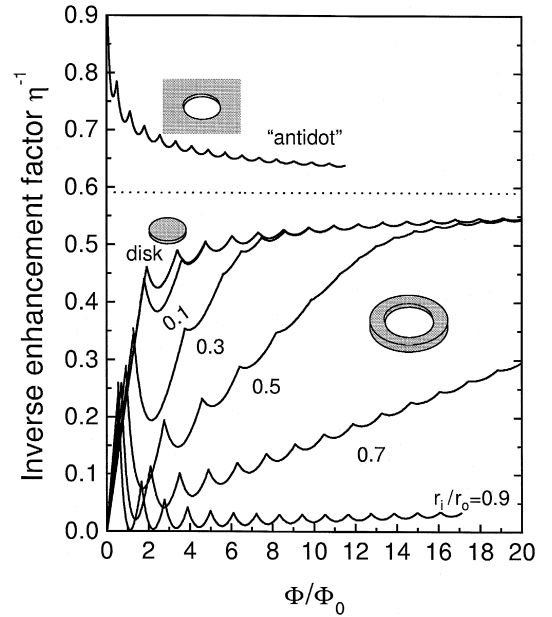


Fig. 2. Inverse enhancement factor $\eta^{-1} = \epsilon(H_{c3}^*)/\epsilon(H_{c2})$ (see Eq. (5)) for loops with different aspect ratio, compared to the case of a disk and an antidot. The horizontal dashed line at $\eta^{-1} = 0.59 = 1/1.69$ corresponds to $H_{c3}(T)/H_{c2}(T) = 1.69$ for a plane superconductor/vacuum boundary [3].

background. This regime can be described within the London limit [14], since superconductivity nucleates almost uniformly within the sample.

In summary, we have solved the linearized GL equation for loops of different wire width, with Neumann boundary conditions at both the outer and the inner loop radii. The critical fields $H_{c3}^*(T)$ are always above the $H_{c3}(T) = 1.69H_{c2}(T)$. The ratio $H_{c3}^*(T)/H_{c2}(T)$ is enhanced most strongly when the sample’s surface-to-volume ratio is the largest. The $T_c(\Phi)$ behaviour can be split in two regimes: for low flux, the background of T_c is parabolic (characteristic for 2D behaviour) and the Little–Parks $T_c(\Phi)$ oscillations are perfectly periodic. In the high-flux regime, the period of the $T_c(\Phi)$ oscillations is decreasing with Φ and the background T_c reduction is quasi-linear (3D regime). The 2D–3D crossover between the two regimes, at a certain applied flux Φ , is similar to the dimensional transition in thin films subjected to a parallel field. As soon the 3D regime is reached, a giant vortex state is created, where only a sheath close to the sample’s outer interface is superconducting.

Acknowledgements

The authors wish to thank H.J. Fink for stimulating discussions. This work has been supported by the Belgian IUAP, the Flemish GOA and FWO-programmes, and by the ESF programme VORTEX.

References

- [1] W.A. Little, R.D. Parks, *Phys. Rev. Lett.* 9 (1962) 9.
- [2] R.D. Parks, W.A. Little, *Phys. Rev.* 133 (1964) A97.
- [3] D. Saint-James, *Phys. Lett.* 15 (1965) 13.
- [4] D. Saint-James, P.-G. de Gennes, *Phys. Lett.* 7 (1963) 306.
- [5] O. Buisson, P. Gandit, R. Rammal, Y.Y. Wang, B. Pannetier, *Phys. Lett. A* 150 (1990) 36.
- [6] V.V. Moshchalkov, L. Gielen, C. Strunk, R. Jonckheere, X. Qiu, C. Van Haesendonck, Y. Bruynseraede, *Nature* 373 (1995) 319.
- [7] H.J. Fink, *Phys. Rev.* 177 (1969) 732.
- [8] H.A. Schultens, *Z. Phys.* 232 (1970) 430.
- [9] B.J. Baelus, F.M. Peeters, V.A. Schweigert, *Phys. Rev. B* 61, in press.
- [10] A. Bezryadin, B. Pannetier, *J. Low Temp. Phys.* 98 (1995) 251.
- [11] A. Bezryadin, A. Buzdin, B. Pannetier, *Phys. Lett. A* 195 (1994) 373.
- [12] V. Bruyndoncx, M. Van Look, M. Verschuere, V.V. Moshchalkov, *Phys. Rev. B* 60 (1999) 10468.
- [13] M. Abramowitz, I.A. Stegun, *Handbook of Mathematical Functions*, Dover, New York, 1970.
- [14] R.P. Groff, R.D. Parks, *Phys. Rev.* 176 (1968) 567.